Injective and surjective machine morphisms

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We investigate one specific three sorted algebra $\langle Q, A, B, \circ, *\rangle$ so called Mealy machine: Q, A, B are finite, nonempty sets; $Q \times A \xrightarrow{\circ} Q$ is a total function and $Q \times A \xrightarrow{*} B$ is a total surjective function.

Definition 1 *Let* $V = \langle Q, A, B\rangle$, $'V = \langle 'Q, 'A, 'B\rangle$ *be machines. We say that* $'V$ simulates V by

$$Q \xrightarrow{h_1} {}'Q, \quad A \xrightarrow{h_2} {}'A^*, \quad {}'B^* \xrightarrow{h_3} B \qquad \textit{if}$$

$q \circ u * a = h_3(h_1(q) \circ h_2(u) * h_2(a))$ *for all* $(q, u, a) \in Q \times A^* \times A$.

Let $T(Q)$ denotes the semigroup of all transformations on the set Q and let $Fun(Q, B)$ denotes the set of all total maps from Q to B. On the set $S(Q, B) = T(Q) \times Fun(Q, B)$ define the multiplication by

$$(\alpha_1, \beta_1)(\alpha_2, \beta_2) = (\alpha_1\alpha_2, \alpha_1\beta_2);\ \alpha_1, \alpha_2 \in T(Q),\ \beta_1, \beta_2 \in Fun(Q, B).$$

Under this operation $S(Q, B)$ is easily seen to be a semigroup.

Let $Q = \{q_1, q_2, \ldots, q_k\}$, $A = \{a_1, a_2, \ldots, a_m\}$, $B = \{b_1, b_2, \ldots, b_n\}$,

$$A \xrightarrow{\alpha} T(Q): \quad a \mapsto \begin{pmatrix} q_1 & q_2 & \cdots & q_k \\ q_1 \circ a & q_2 \circ a & \ldots & q_k \circ a \end{pmatrix},$$

$$A \xrightarrow{\beta} Fun(Q, B): \quad a \mapsto \begin{pmatrix} q_1 & q_2 & \cdots & q_k \\ q_1 * a & q_2 * a & \ldots & q_k * a \end{pmatrix},$$

$$A \xrightarrow{\eta} S(Q, B): \quad a \mapsto \quad (\alpha(a), \beta(a)).$$

The semigroup $\langle V\rangle$ generated by $\eta(A)$ is called the machine V semigroup.

Definition 2 *We say that* $\langle V\rangle \xrightarrow{\psi} \langle 'V\rangle$ *is the s-morphism of the machine semigroup* $\langle V\rangle$ *to* $\langle 'V\rangle$ *if there exist maps* $Q \xrightarrow{g} {}'Q$, $B \xrightarrow{h} {}'B$ *such that the diagram*

$$\begin{array}{ccccc} Q & \xrightarrow{\sigma} & Q & \times & B \\ g \downarrow & & g \downarrow & & \downarrow h \\ 'Q & \xrightarrow{\psi(\sigma)} & 'Q & \times & 'B \end{array}$$

commutes for every $\sigma \in \langle V\rangle$. *If* g *is a surjection the s-morphism* ψ *is called the surjective s-morphism. If* h *is an injection the s-morphism* ψ *is called the injective s-morphism.*

Theorem 3 (i) *If there exists the surjective s-morphism* $\langle V\rangle \xrightarrow{\psi} \langle 'V\rangle$ *then* ψ *is the semigroup* $\langle V\rangle$ *homomorphism to* $\langle 'V\rangle$.

(ii) *If there exists the injective s-morphism* $\langle V\rangle \xrightarrow{\psi} \langle 'V\rangle$ *then* $'V$ *simulates* V.

(iii) *There exists the injective s-morphism* $\langle V\rangle \xrightarrow{\psi} \langle 'V\rangle$ *such that* ψ *is not the semigroups* $\langle V\rangle$, $\langle 'V\rangle$ *homomorphism.*

Implications in sectionally pseudocomplemented posets

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An implication-like operation on a sectionally pseudocomplemented lattice (i.e. lattice with 1 in which every interval $[p, 1]$ is pseudocomplemented) was defined by I. Chajda in [1]. We extend his construction to posets and present a $(\wedge, \vee)$-free axiom system for this operation. We also study some elementary properties of upper semilattices, lower semilattices and lattices equipped with this kind of implication.

References

[1] I. Chajda. *An extension of relative pseudocomplementation to nondistributive lattices.* Acta Sci. Math. (Szeged) **69** (2003), 491–496.

A characterization of some groups of order 2^{2n+1} by their endomorphism semigroups

Tatjana Gramushnjak (Tallinn University),
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For every $n \geq 3$ there are 17 non-isomorphic groups of order 2^{2n+1}, which can be represented in the form $G = (C_{2^n} \times C_{2^n}) \rightthreetimes C_2$, where C_{2^n} and C_2 are cyclic groups of order 2^n and 2, respectively. It is proved, that some of this groups are determined by their endomorphism semigroups in the class of all groups.

Monadic Residuated Algebras

Revaz Grigolia (Tbilisi State University)

Residuated structures appears in many areas of mathematics, the main origin of which are monoidal operation of multiplication $\cdot$, that preserves a partial order, and a binary (left-) residuated operation $\rightarrow$ characterized by $x \cdot y \leq z$ if and only if $x \leq y \rightarrow z$. Such kind of structures are associated with logical systems. If the partial order is a semilattice order and multiplication the semilattice operation, we get Browerian semilattices which are models of the conjucture-implicatiom fragment of the intuitionistic propositional calculus. The well-known algebraic models of the conjucture-implicatiom fragment of Lukasiewicz many-valued logic are another example of special class of residuated structures. We are interested mainly with those monoidal structures which have in common the following basic properties: Integrality, commutativity of the monoidal operation $\cdot$ and the existence of a binary operation $\rightarrow$ which is adjoint to the given operation $\cdot$.

We represent three type of monadic algebras: monadic Heyting algebras, monadic MV-algebras and monadic BL-algebrs, which represent, respectively, monadic Intuitionistic logic, monadic Lukasiewicz logic and monadic Basic logic. The monadic algebras are represented as a pair of algebras one of which is special kind of relatively complete subalgebras.

Generators in the category of S-posets

Valdis Laan (University of Tartu)

We give some characterizations of generators and cyclic projective generators in the category of ordered right acts over an ordered monoid.

Quantifiers on quantale algebras

Sergejs Solovjovs (University of Latvia)

The classical notion of quantifier on a Boolean algebra introduced by P. Halmos (1955) induced many researchers to study various generalizations of the notion. In particular, A. Monteiro and O. Varsavsky (1957) considered the concept for Heyting algebras, J. D. Rutledge (1959) for MV-algebras, but M. F. Janowitz (1963) used orthomodular lattices. Following the general move as well as in continuation of our study of the category Q-**Alg** of algebras over a given unital commutative quantale Q, we introduce quantifiers on quantale algebras. We are interested in characterization of certain kind of monomorphisms of the category Q-**Alg** through the set of quantifiers on a fixed Q-algebra A. We also show a representation theorem for quantifiers through generalized equivalence relations.

Monadic bounded commutative residuated ℓ-monoids

Filip Švrček (Palacký University, Olomouc)

Bounded commutative residuated ℓ-monoids ($R\ell$-monoids) are generalizations of residuated algebras of propositional logics such as BL-algebras, i.e. algebraic counterparts of the basic fuzzy logic (and hence consequently MV-algebras, i.e. algebras of the Łukasiewicz infinite valued logic) and Heyting algebras, i.e. algebras of the intuitionistic logic. Monadic MV-algebras are algebraic models of the predicate calculus of the Łukasiewicz infinite valued logic in which only a single individual variable occurs. The monadic $R\ell$-monoids will be introduced as generalizations of monadic MV-algebras and the results done for the monadic MV-algebras as e.g. the results related to subdirect irreducibility will be discussed for the class of monadic $R\ell$-monoids.

Notes on Morita duality for S-posets

Lauri Tart (University of Tartu)

In the previous seminar it was shown that Morita equivalence for S-posets follows the same pattern as Morita equivalence for S-acts and provides a near-isomorphism relation. As an attempt to weaken this realation, we study what can be transferred from the theory of Morita duality for S-acts to form the Morita duality of S-posets. These first notes shall deal with the issues of finding whether and how it makes sense to define Morita duality for S-posets, primarily dealing with the properties and use of Morita subcategories.