

# Free Planar Graphs on Torus

Dainis ZEPS

Institute of Mathematics and Computer Science  
University of Latvia, 29 Rainis blvd., Riga, Latvia  
dainize@cclu.lv

Graph on plane, projective plane or torus is free planar if it remains embeddable in corresponding surface if arbitrary edge is added.

Free planar graph is called *ladder-augmentable* if two edges in it may be replaced with arbitrary large ladder-graph[?] and graph will remain to be free planar with respect to surface where it is embedded.

There are only two nontrivial 3-connected free-planar graphs on sphere, i.e. wheel graph and envelope graph, where only the first can be unboundedly augmented with a *punkah* like augment[?] retaining its 3-connectivity and freeness on the sphere. Envelope graph may be augmented with ladder like augment only when it is on the Mobius strip.

**Theorem 1.** *If 3-connected toroidal free planar graph is ladder-augmentable then it should be projective planar.*

Theorem higher does not exclude case when graph is toroidal free planar and projective planar but it is not projective free planar.

$K_{3,3}$  and  $K_5$  are ladder-augmentable because they both are projective-planar[?]. Similarly  $4*4$ -lattice on projective plane, which is minimal forbidden minor for torus [?]. In fig.1, this same graph with two eliminated horizontal edges, becomes toroidal free planar. Toroidal freeness is checked, using computer. But it is not projective free planar[?].

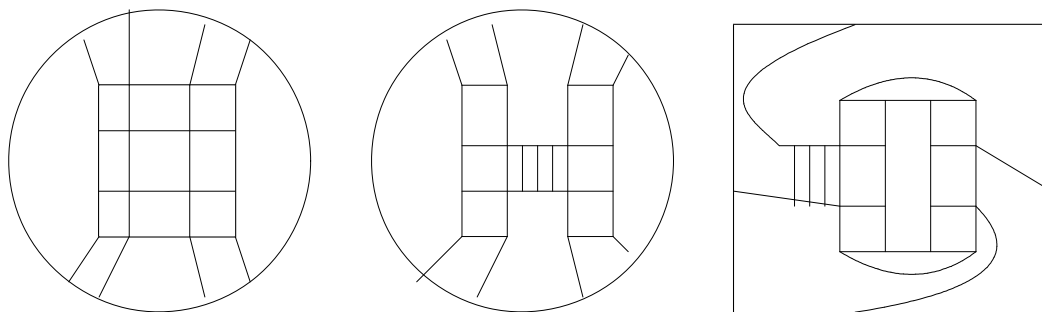


Figure 1:  $4*4$  lattice [minus two edges and augmented] on projective plane and torus.

Example of *punkah* augmentable graph[?] that is not projective planar is shown in [?].

## References

- [1] B. Mohar, C. Thomassen. *Graphs on surfaces*, The John Hopkins University Press, Baltimor and London, 2001.
- [2] D. Zeps. *Free Planar Graphs on Torus: checking 3-connected graphs for unbounded augmentability*, to be published, 5pp, see <http://www.ltn.lv/dainize/MathPages/unikonf2004.pdf>