

Four levels of complexity in mathematics and physics

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Abstract

Four levels of complexity in mathematics and physics are considered, how they are interrelated, how this all has impact on other subjects of epistemology.

Key words: ToE, E8, octonions, four color theorem, particle wave duality, particle space duality, languages, cognition

Introduction

Mathematics most easy may discern, or, at least, try to discern, four levels of complexity. According a four level argument, most easy and firstly may be classified real numbers, complex numbers, quaternions, octonions. Next available application of such argument could be Lie group classification: last would be E8, on third level might stand E8 root system (Gosset 4 21 polytope), on second – G2 and F4, and on first – simple Lie groups. May mathematics in general be classified according four level complexity argument? We don't know, but we may try to get to the reasonable point of the argument trying all this in connection via physics, and then after with other subjects of epistemology.

Using four level complexity argument in physics, we may put the matter and forces, arranged in ToE on level four. Then after, on level three would come the matters decomposition into elementary particles. On second level might be put particle interaction schemes according particular patterns, that should give us four fundamental forces of nature. On first base 4-manifold of space-time may be assigned.

Was Maxwell right?

James Clerk Maxwell didn't believe in atomic structure of matter. Was he right? Do atoms, particles, quarks exist? Positivists give answer in their positivistic style, saying that the question does not have sense, because all should decide experiment, and the answers

should be given in the language of measurements. Well. Let it be so. Mathematics is what (or who) says yes or no to all questions according its own inner logic and compatibility, in physics in particular and in general too. But what we register in particle accelerators? What we see in camber of Wilson? What we classify in SM? What we predict via BB? The ultimate answer may come in some form similar to Garrett Lisi theory of everything (1). This theory is up to now under question about its provability. But the fact itself that such theory has appeared is signaling that maybe Maxwell was right. If the general form of matter and general force accordingly has the form E8 then this may say in other words that particles are only sort of illusion, that we see something as such on some levels of complexity, where on the most general level of complexity for all this stand one general force in form of E8 symmetry with matter in form of E8 symmetry, where we do not now have any sense to speak about matter and force separately, but only say, that nature has form of E8. Period. This now looks like real theory of everything, and first who predicted this was James Clark Maxwell. Well, of course, at first this is very figurative description, but taking into account, that Maxwell gave a first pattern of this general force in the form of his famous equations, we may attribute to Maxwell earliest fame of the theory of everything. Moreover, speaking about ToE has sense only in this form predicted by Maxwell about non atomic structure of the matter. If we try to argue that he, Maxwell, wasn't very successful in trying to apply his approach to gravity, then this is due to the fact that E8 was found only in 1890 and developed to a state to give first proposal to ToE only in 2007, i.e., almost 130 years after his death, and he didn't know that he already gave maximal possible contribution to ToE, and was still working, in place to receive maximal deserved applause.

It may be useful to remember another thinker who may be mentioned with some insight in the future further than people around him, e.g., Newton, and this man was George Berkeley. Of course, we may make excuses that particles were only some convenient way to speak about structure of matter, and we blamed Maxwell on his non-atomism only in some friendly manner of familiarity, but if there are people who say directly that we tend to think in manner following our sensuality and that this sensuality may have affected our actual possibility to argue scientifically at all, then such people should be taken more than seriously, and one of such man was George Berkeley. According Berkeley or his argument in today's statement, our tendency to think of matter as consisting from particles was only some sensual approach to interpret physical reality, and 'dropped stone on leg' argument in Berkeley's time turns into 'electron beam on screen' argument today, but argument of what is reality remains the same. Reality is somewhere behind all what we perceive – Berkeley's argument may turn to be most acceptable. For Berkeley behind all was standing God. For us today we have in place of God in this sense mathematics, but it is only for a while, maybe, see (2). Author argues, that what actually stands behind all, what is ultimate reality, is life, principle of life, *vita principalis* (2). What gave a persuasion to Berkeley so fiercely attack Newton on his absolute time conception? Newton was after all right. But Berkeley was right too, but from another point of view. Excluding what may be excluded as a result of perception, Berkeley stepped back more and more, finding in the background of all the creator God, but this was only first step. After descending by gradual steps on secondary order forms of reality, he found reality that caused all these levels, but then he turned in time (Pythagorean ladder according Einstein) and did the same in time, and came to a point

where God stood as the creator of all this. Time according Berkeley, acquired starting point, but he did not this simply following biblical argument as a priest and as a bishop, but as a scientist, that gave him personal right to argue on this point with the most famous scientist of the time – Newton. We may call Berkeley's used argument against Newton Berkeley's argument, and find this same argument being used in SM and BB connection. Again, one uses God in place where in other approach mathematics now is standing.

Four level complexity in mathematics

We started argument with number systems: real, complex, quaternion, octonion. See (3). Further we go in repetitions. R, C, H, O – these are divisional algebras, and only. These are basic in Clifford algebra, except that O is not Clifford algebra, but plays fundamental role in them. According Baez (3), first eight Clifford algebras $\text{Cliff}(n)$ are: $\text{Cliff}(0) \rightarrow \mathbb{R}$, $\text{Cliff}(1) \rightarrow \mathbb{C}$, $\text{Cliff}(2) \rightarrow \mathbb{H}$, $\text{Cliff}(3) \rightarrow \mathbb{H} \oplus \mathbb{H}$, $\text{Cliff}(4) \rightarrow \mathbb{H}[2]$, $\text{Cliff}(5) \rightarrow \mathbb{C}[4]$, $\text{Cliff}(6) \rightarrow \mathbb{R}[8]$, $\text{Cliff}(7) \rightarrow \mathbb{R}[8] \oplus \mathbb{R}[8]$. Further come repetitions according formula $\text{Cliff}(n+8) \cong \text{Cliff}(n) \oplus \mathbb{R}[16]$. But irreps (irreducible matrix representations) for $\text{Cliff}(n-1)$ are only for $n=1,2,4,8$, thus, again “mystical” four.

We know famous Four Color Theorem (4). It may turn out to be the most complex theorem, what concerns its proof, among simplest theorems, what concerns their formulation. At level four we see unexplainable complex case, where all other cases are incomparably simpler, see (5). In (6) we show that there four levels of complexity are discernable as follows: null level (e.g. outer planar graphs), tetrahedral graphs, free-planar graphs, planar graphs.

Quantum mechanics as a first level of complexity

At null level something without quantum picture should stand. Second order complexity may be quantum field theory or Standard Model. Third – ToE in an outline how we tried to show higher, i.e., as general field of nature, as E8 or other, for future to decide.

At level one, quantum mechanics as particle wave equivalence would stand. Particle space equivalence would be of second order complexity if such an approach came to implementation (7).

Complexity invariance principle

Let us formulate a principle according which we are built in the same complexity which we get as an ability to act. For human beings this ability is on comprehension level and we are proud to comprehend that we have the same level of complexity comprehension as we are created, at least what concerns our reference system of life. Of course it may be only illusion and world of much higher level of complexity is somewhat prohibited for us, but we, together with Plato, express hope, that “soul comprise all universe”, and we may use our

complexity invariance principle saying: We are as complex as it is given in our comprehension to comprehend. One would say that this is tautology. Maybe, but using this “tautology” we have come up to quarks and ToE, and Plato up to universe.

But this same principle says something more, and this is a level of complexity of comprehension, namely, four. Actually, number four is hypothetical, maybe five or even seven must stand for it. The crucial is the fact that this level of complexity is so small. Because of this smallness we are holding as if to the minimum possible value of it, i.e., four, not seven, say. What we actually are going to say is that this level of complexity given to us is not lower than four.

Maximal complexity subsystems

For us it is crucial to comprehend that maximal complexity cognition subsystems exist all over us, in whatever our scope of activities. First should be mentioned languages. Languages are as complex as we can them have maximally. It can be seen at least from the fact that different languages are of the same complexity, all of them. Higher level comprehension is not given to us principally. In languages, in linguistics, where it is harder to distinguish, similarly as in mathematics, four levels of complexity. Why? Because the comprehension itself via language, via notion comprehension mechanism is on maximal level of complexity accessible for us, i.e., comprehension via language already uses maximal level of complexity, thus, level four. In (8) we tried to show this via theorem windows notion which is built from four singularities. All what we can see in languages is that four level (or at least low level) complexity suits mostly to describe morphological and other phenomena in them.

More crucial point is that complexity of language is the same that of the comprehension in general, even more, that language isn't distinguishable from cognition, what Benjamin Lee Whorf was arguing in (9).

One more necessary aspect we need, namely, different maximal complexity subsystems can't be mixed into themselves. What does this mean? If we would try to mix two high level complexity subsystems in one subsystem, resulting subsystem would become incredibly complex, but we can comprehend only four level complex subsystem. Mathematicians would argue that we may mix, using appropriate techniques, arbitrary system in new systems of arbitrary complexity. But it is only seemingly leading way to some new solution. First, such complex systems would be not manageable, they we would break in simpler subsystems in any way, or would remain unsolvable to us.

As a good example we have the same for languages. We never mix them up but use them how we have taken them, Latin within Latin, Greek within Greek. Using the same pattern we should work in other fields of cognition too.

Human experience samples as maximal complexity subsystems

Good examples of maximal complexity subsystems are human experience systems. First of we would like to mention these human experiences that deserve to exist as such first of all, namely, most famous experiences of greatest philosophers and so on.

We would single out most crucial aspect of our approach. Namely, most famous human experiences must be considered as maximal complexity subsystems for one practical reason. They can't be mixed into some oversystems, or supervisory systems. But this is what the contemporary science is doing all way around. We build philosophical supervising systems, then we come to comprehend that philosophy has come to its end as some philosophers try to state this. Why? Because they are working with broken subsystems. If we make a common system from Aristoteles and Plato, we already get a system, where Plato and Aristoteles are broken, because we can't manage adequately system with higher level of complexity than we may have. Because Plato and Aristoteles already are maximal subsystems and we can't make then into a common system without breaking them as subsystems. Philosophers are trying to keep Plato and Aristoteles unbroken? Maybe in times of Schopenhauer, when he first, before starting to study Plato, translated all of him into German. Nowadays we are content with a broken Plato, with a broken Aristoteles, and so on.

In order to regain scientific argument in the science, we had to reconsider today all about thinkers of past. We should leave aside our self-assumption when we say, e.g., that we are cleverer than Plato, saying, Plato didn't know what we use to know today. It is nonsense, to argue in this way. We lose Plato, we lose all before Plato, we fill our as if scientific argument with heaps of debris from broken experiences of the thinkers of the past.

Conclusions

We may list some maximal complexity subsystems:

1. whatever cognitive activity;
2. notion building activity (8);
3. theorem window (8);
4. language (9);
5. human lifelong experience;
6. level of complexity in mathematics (10) ;
7. level up to what we may discern nature;
8. highest possible level of complexity of whatever science.

Contemporary sciences, how things are judged by the author, mostly are on the first level of complexity. Best of all it is seen in philology, biology, theory of evolution, as well as in all phenomenal sciences which are not using deduction in analogy with mathematics and physics. Exceptions are mathematics itself, and theoretical physics that has reached at least level three (with SM and BB) and is now on a way to the fourth level (e.g., ToE). On higher level of complexity are religious systems, mostly on fourth level (11; 12; 13; 14; 15; 16; 17;

18; 19; 20). Religious way of thinking has acquired much higher amount of experience than whatever science, and this experience shouldn't be neglected in any way. Thus, "theories" about dark medieval times should be removed from our thinking in order to overcome "dark times" in contemporary sciences.

In the end we would like to say that nature, or God, has granted for *homo sapiens* cognitive activity that is maximal complexity level for human beings already, but in sciences we have reached not far from the first level. According our hypothesis we have maximal level of cognitive complexity that is maximal possible in sense we are built using this same level of complexity (2; 21). Plato, Aristoteles, Descartes, and many other thinkers of past were on fourth level, but our sciences can't step further from the first level. Exception is the theoretical physics, thus, the physics is not at all in trouble but the leading one of all other sciences (22; 23; 24; 25; 26; 27; 28; 29; 14; 30) (31; 32; 33; 34), [see also (11; 35; 36; 13)], only physicists themselves are weak in the recognition of this fact (37; 38; 39).

References

1. **Lisi, A. Garrett.** *An Exceptionally Simple Theory of Everything*. 2007. p. 31. arXiv:0711.0770v1.
2. **Zeps, Dainis.** *Mathematics as Reference System of Life*. Riga : Internet publication, 2009.
3. **Baez, John C.** *The Octonions*. Riverside : University of California, 2001.
4. **Thomas, Robin.** *An Update on the Four-Color Theorem*. s.l. : Notices of the AMS, 1998. Vol 45, Num 7.
5. **Ringel, Gerhard.** *Map Color Theorem*. NY : Springer, 1974.
6. **Zeps, Dainis.** *Application of the Free Minor Closed Classes in the Context of the Four Color Theorem*. Riga : Quantum Distinctions, 2007. <http://www.ltn.lv/~dainize/idems.html>.
7. —. *Space particle duality*. 2008.
8. —. *Building Mathematics via Theorem Windows*. Riga : Quantum Distinctions, 2009. <http://www.ltn.lv/~dainize/idems.html>.
9. **Whorf, Benjamin Lee.** *Language, Mind and Reality*. 1952. lpp. Vol. IX, No 3, 167-188.
10. **Wigner, E.** *The unreasonable effectiveness of Mathematics in the natural science*. 1960. pp. 1-14. www.math.ucdavis.edu/~mduchin/111/readings/hamming.pdf.
11. **D'Aquili, Eugene and Newberg, Andrew B.** *The Mystical Mind: Probing the Biology of Religious Experience*. s.l. : Augsburg Fortress Publishers, 1999.

12. **Ouspensky, Peter.** *Tertium Organum. Key to Solving Mysteries of the World. In Russian.* 1911.
13. **Steiner, Rudolf.** *Die vierte Dimension. Mathematik und Wirklichkeit.* Dornach : R. Steiner Verlag, 1995.
14. **Tipler, Frank J.** *Structure of the World from Pure Numbers.* 2008. pp. 897-964. arXiv:0704.3276v1.
15. —. *The Physics of Immortality.* s.l. : Doubleday, 1994.
16. —. *The Physics of Christianity.* s.l. : Doubleday, 2007.
17. **Zeps, D.** *Cognitum hypothesis and cognitum consciousness. How time and space conception of idealistic philosophy is supported by contemporary physics.* 2005.
18. **Vernadski, Vladimir.** *Biosphere and Noosphere (In Russian).* Moscow : s.n., 2002.
19. **Zeps, D.** *Classical and Quantum Self-reference Systems in Physics and Mathematics .* 2007.
20. **Zeps, Dainis.** *Rudolf Steiner on mathematics and reality. In Latvian.* 2008. p. 7 pp.
21. —. *Cogito ergo sum.* 2008.
22. **Huang, Kerson.** *Fundamental Forces of Nature. The Story of Gauge Fields.* Singapore : World Scientific, 2007.
23. **Penrose, Roger.** *The Road to Reality. A Complete Guide to the Laws of the Universe.* New Yourk : Vintage Books, 2007.
24. **Bohm, David.** *Wholeness and the Implicate Order.* London : Routledge, 2002.
25. **Rashewsky, Peter.** *Rieman Geometry and Tensor Analysis. In Russian.* 1967.
26. **Smolin, Lee.** *Three Roads to Quantum Gravity.* New Yourk : Basic Books, 2001.
27. **Hall, Brian C.** *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction.* New Yourk : Springer, 2003.
28. **Tegmark, Max.** *Is 'the theory of everything' merely the ultimate ensemble theory?* 1998. arXiv:gr-qc/9704009v2.
29. —. *Mathematical Universe.* 2007. arXiv:0704.0646v2.
30. **Vladimirov, J. S.** *Geometrofizika. In Russian.* M. : s.n., 2005.
31. **Weinberg, Steven.** *The First Three Minutes: A Modern View of the Origin of the Universe.* s.l. : Basic Books, 1993.
32. **Woit, Peter.** *Not Even Wrong. The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics.* London : Jonathan Cape, 2007.

33. **Zeps, D.** *On to what effect LHC experiments should arrive.* 2007.
34. **Zeps, Dainis.** *Hologram and distinction.* 2008.
35. **Dlyasin, G.** *Azbuka Germesa Trismegista ili molekularnaja tainopis mishelnija.* 2002.
36. **Lakoff, George and Nunez, Rafael E.** *Where mathematics comes from. How the embodied mind brings mathematics into being.* s.l. : Basic Books, 2000.
37. **Smolin, Lee.** *The Trouble with Physics. The Rise of String Theory, the Fall of a Science and What Comes Next.* s.l. : A Mariner Book, 2006.
38. **Zeps, Dainis.** *The trouble with physics. How physics missed main part of the observer and what comes next.* 2008. p. 9.
39. —. *Trouble with physical interpretations or time as aspect of reference system of life.* 2008.