

# Four levels of complexity in mathematics and physics

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## **Abstract**

Four levels of complexity in mathematics and physics are considered, how they are interrelated, how this all has impact on other subjects of epistemology.

**Key words:** ToE, E8, octonions, four color theorem, particle wave duality, particle space duality, languages, cognition

## **Introduction**

Mathematics most easy may discern, or, at least, to try to discern, four levels of complexity. According four level argument, first may be classified real, complex, quaternion, octonion numbers. The next necessary application should be Lie group classification: last should be E8, on third level may pretend E8 root system (Gosset 4 21 polytope), on second – G2 and F4, and on first – simple Lie groups. May mathematics in general be classified according four level complexity argument? It may be hardly arguable, but we try to get to the reasonable point of the argument trying all this in connection via physics, and then after with other subjects of epistemology.

Using four level complexity argument in physics, we may put matter and forces, arranged in ToE on level four. Then after, on level three comes matters decomposition in elementary particles. On second level may be put particle interaction schemes according particular patterns, that should give us four fundamental forces of nature. On first base 4-manifold of space-time may be assigned. Accordingly we may try to classify forces' arrangement arising from gauge freedom. General force of nature, connection? ..., four fundamental forces, connection? ..., according this maybe pattern, or like, reconstruction of four levels should be attacked.

## Was Maxwell right?

James Clerk Maxwell didn't believe in atomic structure of matter. Was he right? Do atoms, particles, quarks exist? Positivists give answer in their positivistic style, saying that the question does not have sense. Well. Let it be so. Mathematics is what (or who) says yes or no to all question, in physics in particular and general too. But what we register in particle accelerators? What we see in chamber of Wilson? What we classify in SM? What we predict via BB? The ultimate answer may come in some form similar to Garrett Lisi theory of everything (1). This theory is up to now under question about its arguability. But the fact that such theory has appeared is signaling that maybe Maxwell was right. If general form of matter and general force accordingly has form E8 then this may say in other words, that particles (and interactions as particles, truth both fermions and bosons) was only illusion, that we see something as such on some levels of complexity, on most general level of complexity for all this standing one general force in form of E8 symmetry with matter in E8 symmetry, where we now do not have any sense to speak of matter and force separately, but only say, that nature has form of E8. Period. This now looks like real theory of everything, and first who predicted this was James Clark Maxwell. Well, of course, first this is very relative designation, but taking into account, that Maxwell gave first pattern of this general force in form of his fames equations, we may attribute to Maxwell earliest fame of the theory of everything. Moreover, speaking about ToE has sense only in this form, predicted by Maxwell about non atomic structure of matter. If we try to argue that he, Maxwell, wasn't very successful in trying to apply his approach to gravity, then this due to fact, that E8 was found only in 1890 and developed to state to give first proposal to ToE only in 2007, i.e., after his death, and he didn't know that he already gave maximal possible contribution to ToE, and was still working in place to receive maximal deserved applause.

It may be useful to remember another thinker, who may be mentioned with some insight in future further than people around, e.g., Newton, and this man was George Berkeley. Of course, we may make excuses, that particles were only some convenient way to speak about structure of matter, and we blamed Maxwell on his non-atomism only in some friendly manner of familiarity, but if there are people which say directly that we tend to think in manner following our sensuality and that this sensuality may affect our actual possibility to argue scientifically at all, then such people should be taken more than seriously, and one such man is George Berkeley. According Berkeley or his argument in today's statement, our tendency to think of matter as consisting from particles was only some sensual approach to interpret physical reality, and 'dropped stone on leg' argument in Berkeley's time turns into 'electron beam on screen' argument today, but argument of what is reality remains the same. Reality is somewhere behind all what we perceive – Berkeley's argument may turn to be most acceptable. For Berkeley behind all was standing God. For us in today we have in place of God in this sense mathematics, but it is only for a while, maybe, see (2). Author argues, that what actually stands behind all, what is ultimate reality, is life, principle of life, *vita principalis* (2). What gave persuasion to Berkeley so fiercely attack Newton on his absolute time conception? Newton was after all right. But Berkeley was right too, but from another point of view. Excluding what may be excluded as result of perception, Berkeley stepped back more and more, finding in the background of all the creator God, but this was only first step. After descending in gradual steps on secondary order forms of reality, he

found reality what caused all these levels, but then he turned in time (Pythagorean ladder according Einstein) and did the same in time, and came to point where God stood as creator of all this. Time according Berkeley, acquired starting point, but he did not this simply following biblical argument as priest and as bishop, but as scientist, that gave him personal right to argue on the point with the most famous scientist of the time – Newton. We may call Berkeley's used argument against Newton Berkeley's argument, and find this same argument being used in SM and BB connection. Again, one uses God in place, where in other approach mathematics now is standing.

#### **Four level complexity in mathematics**

We started argument with number systems: real, complex, quaternion, octonion. See (3). Further we go in repetitions. R, C, H, O – these are divisional algebras, and only ones. These are basic in Clifford algebra, except that O is not Clifford algebra, but plays fundamental role in them. According Baez (3), first eight Clifford algebras  $Cliff(n)$  are:  $Cliff(0) \rightarrow R$ ,  $Cliff(1) \rightarrow C$ ,  $Cliff(2) \rightarrow H$ ,  $Cliff(3) \rightarrow H \oplus H$ ,  $Cliff(4) \rightarrow H[2]$ ,  $Cliff(5) \rightarrow C[4]$ ,  $Cliff(6) \rightarrow R[8]$ ,  $Cliff(7) \rightarrow R[8] \oplus R[8]$ . Further come repetitions according formula  $Cliff(n+8) \cong Cliff(n) \oplus R[16]$ . But irreps (irreducible matrix representations) for  $Cliff(n-1)$  are only for  $n=1,2,4,8$ , thus, again “mystical” four.

We know famous Four Color Theorem (4). It may turn to be most complex theorem, what concerns its proof, among simple theorems, what concerns its formulation. At level four we see unexplainable complex case, where all other cases are incomparably simpler, see (5). In (6) we show that there four levels of complexity are discernable as follows: null level (e.g. outer planar graphs), tetrahedral graphs, free-planar graphs, planar graphs.

#### **Quantum mechanics as first level of complexity**

At null level something without quantum picture should stand. Second order may be quantum field theory or Standard Model. Third – ToE in an outline how we tried to show higher, i.e., as general field of nature, as E8 or other, for future to decide. To use this picture, quantum mechanical interpretations should be developed accordingly:

At level one, quantum mechanics as particle wave equivalence is developed. Particle space equivalence would be second order complication if this approach may come into implementation (7).

#### **Complexity invariance principle**

Let us formulate principle according which we are built in the same complexity which we get as ability to act. For human beings this ability is on comprehension level and we are proud to comprehend that we have the same level of complexity comprehension as we are created, at least what concerns our reference system of life. Of course it may be only illusion and

world of much higher level of complexity is somewhat prohibited for us, but we, together with Plato, express hope, that “soul comprise all universe”, and we may use our complexity invariance principle saying: We are as complex as is given in our comprehension to comprehend. One would say that this is tautology. Maybe, but using this “tautology” we have come up to quarks and ToE, and Plato up to universe.

But this same principle says something more, and this is level of complexity of comprehension, namely, four. Actually, number four is hypothetical, maybe five or even seven must stand for it. The crucial is the fact that this level of complexity is so small. Because of this smallness we are holding as if to the minimum eventual value of it, i.e., four, not seven, say. What we actually say is that this level of complexity given us is not lower than four.

### **Maximal complexity subsystems**

For us it is crucial to comprehend that maximal complexity cognition subsystems exist all over us, in whatever our scope of activities. First should be mentioned languages. Languages are as complex as we can have maximally. Higher level comprehension is not given to us at all. In languages, in linguistics, it is hard to distinguish, similarly as in mathematics, four levels of complexity. Why? Because the comprehension itself via language, via notion comprehension mechanism is on maximal level of complexity, i.e., comprehension via language already uses maximal level of complexity, thus, level four. In (8) we tried to show via theorem windows notion which is built from four singularities. All what we can see in languages, that four level (or at least low level) complexity suits mostly to describe morphological and other phenomena in them.

More crucial point is that complexity of language is the same what of comprehension in general, that language isn't distinguishable from cognition what Benjamin Lee Whorf was arguing, see (9).

One more necessary aspect we need, namely, different maximal complexity subsystems can not be mixed into themselves. What does this mean? If we try to mix to high level complexity subsystems in on subsystem, resulting subsystem would become incredibly complex, but we can comprehend only four level complex subsystem. Mathematicians would argue that we may mix, using appropriate techniques, arbitrary system in new systems of arbitrary complexity. But it is only seemingly leading way to solution. First, such complex systems would be not manageable, they we would break in simpler subsystems in any way.

As good example we have the same languages. We never mix them but use them how we have them, Latin within Latin, Greek within Greek. Using the same pattern we should work in other fields of cognition too.

## **Human experience samples as maximal complexity subsystems**

Good example of maximal complexity subsystems are human experience systems. Here we would like to consider these human experiences that deserve to exist as such first of all, namely, most famous experiences of greatest philosophers and so on.

We would single out most crucial aspect of our approach. Namely, most famous human experiences must be considered as maximal complexity subsystems for one practical reason. They can't be mixed into some oversystems, or supervisory systems. But this is what contemporary science is doing all way around. We build philosophical supervising systems, then we come to comprehend that philosophy comes to its end as some philosophers state this. Why? Because they are working with broken subsystems. If we make system from Aristoteles and Plato, we already get system, where Plato and Aristoteles are broken, because we can't manage system with higher level of complexity than we may have. Because Plato and Aristoteles already are maximal subsystems and we can make then in one common system without breaking them as subsystems. Philosophers are trying to keep Plato and Aristoteles unbroken? Maybe in times of Schopenhauer, when he first, before studying Plato, translated all him into German. Nowadays we are content with broken Plato, with broken Aristoteles, and so on.

In order to regain scientific argument in science, we had to reconsider today all about thinkers of past. We should leave aside our self-assumption when we say, e.g., that we are more clever than Plato, saying, Plato didn't know what we use to know today. It is nonsense, to argue in this way. We loose Plato, we loose before Plato, we fill our as if scientific argument with heaps of debris from broken experiences of the thinkers of the past.

## **Conclusions**

We may list some maximal complexity subsystems:

1. whatever cognitive activity;
2. notion building activity (8);
3. theorem window (8);
4. language (9);
5. human lifelong experience;
6. level of complexity in mathematics (10) ;
7. level up to what we may discern nature;
8. highest possible level of complexity of whatever science.

Contemporary sciences, how things are appreciated by author, mostly are on first level of complexity. Best of all it is seen in philology, biology, theory of evolution, as well as in all phenomenal sciences which are not using deduction in analogy with mathematics and physics. Exceptions are mathematics itself, and theoretical physics that has reached level three (with SM and BB) and is now on way to fourth level (with ToE). On higher level of complexity are religious systems, mostly on fourth level (11; 12; 13; 14; 15; 16; 17; 18; 19; 20). Religious way of thinking has acquired much higher amount of experience than

whatever science, and this experience shouldn't be neglected in no way. Thus, "theories" about dark Medieval times should be removed from our thinking in order to overcome "dark times" in contemporary sciences.

In end we would like to say that nature, or God, has granted for *homo sapiens* cognitive activity that is maximal complexity level for human beings already, but in sciences we have reached not far from first level. According our hypothesis we have maximal level of cognitive complexity that is maximal possible in sense we are built using this same level of complexity (2; 21). Plato, Aristoteles, Descartes, and many other thinkers of past were on fourth level, but our sciences can't step further from first level. Exception is theoretical physics, thus, physics is not at all in trouble but leading all other sciences (22; 23; 24; 25; 26; 27; 28; 29; 14; 30) (31; 32; 33; 34), [see also (11; 35; 36; 13)], only physicists themselves are weak in recognizing this fact (37; 38; 39).

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