The Double Rotation as Invariant of Motion in Quantum Mechanics

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Abstract

Quantum mechanics may loose its weirdness if systematically geometric algebra methods would be used more. Crucial aspect is to find laws of quantum mechanics be present in macroworld in form of description of motions rather than objects. To help to reach this goal we suggest to use double rotation as one of base invariants in quantum mechanics. We suggest to consider geometric algebra as algebra of motion and double rotation specifically.

Key words: quantum mechanics, geometrical algebra, rotation, double rotation, reflection, cognition, reference of life

Introduction

Both in mathematics and physics we tend to consider more readily objects than motions. Objects are what we see and can perform experiments with notwithstanding that contemporary science is capable to do all this to what concerns motions as well. Crucial turning point was discovery of quantum mechanics where object and motion were put on equal footing: bosonic (motion) world is on similar conditions in quantum mechanics as fermionic (object) world. But why quantum mechanics still remains for us as if weird science? Quantum mechanics in its essence tends to persuade us that worlds of motions and objects are not to be discriminated but considered as equivalent, as complementary, as parts of one common reality. Didn't arise some hidden discrimination between objects and motions due to fact that motion we connect too closely with what we may perceive as "something moving" in visible space and time? If so, where to find this more general motion that in microworld reveals to us sufficiently clearly by laws of quantum mechanics at least in fermionic bosonic world picture, but in macroworld do not want to show up with similar ease? David Bohm (1) persuades us to think in terms of holomovement, but according his own statements his holomovement is just that part of reality that is neither perceivable nor measurable (locally) remaining part of reality that pertains to global (nonlocal) aspects of reality and thus, using some free language, is maybe acceptable with ease to Gods but not to us humans.

In this paper we suggest to introduce motion not as something perceivable in act of observation or physical experiment, but as quantity of deduction. In that case we must have something where from to deduce this motion, and we choose for this reason quantum mechanical interpretation introduced in papers (2; 3; 4). In this our attitude we argued that quantum distinction is freedom of motion, where freedom we use in the sense gauged freedom (5; 6; 7). Now let us assume that we take as ground notion quantum distinction, and motion is what is deduced from there, i.e., motion should be taken that what gauged gives us quantum distinction. What may be gained from such approach? Very simply: as quantum distinction we take physical laws already discovered in physical science and motion in general is that what gauged should give these laws. Let us denote motion introduced in this way by Motion, i.e., writing it using capital letter. In this way we try to uncouple motion from whatever space or time aspect, because Motion should be aspect of physical laws discovered in written in mathematical way. What is behind this Motion we may be not too specific saying that it do not bother us similarly as positivistic science doesn't bother about why physical law uses mathematical apparatus in that or other way.

Physical laws as freedom of Motion

In (2; 3; 4) we introduced principle that quantum distinction is freedom of motion. Let us put in place of motion general notion Motion where the last should be deduced from this statement: under Motion we understand (mathematical and physically interpretable) quantity what gauged should give us perceivable reality in form of quantum distinction. But among all what we could call quantum distinction (or distinctions in plural) we have distinction that we call physical laws, let us choice among them the laws that pertain to quantum mechanics. Remember, we choose quantum mechanics because it "knows" how to lower discrimination of motion against object.

What we should gain? Let us try to find at least one general invariant that would pretend to be independent from direct space time aspect in whatever appearance. We argue that we succeed to find one, and that should be the *double rotation*. In order to see this we should turn to Clifford algebra, the algebra of quantum mechanics. It is easy to see that there some tautology are likely to appear: we try to exclude space time aspect, but speak about rotations, namely, double rotation which is hardly to perceive as something uncoupled from space and time aspects. But actually we are going to speak in language of geometric algebra or Clifford algebra where behind what we call rotation should stand operator and operational relation.

The Clifford algebra and rotations

In order to come to main point of our article the double rotation we should consider Clifford algebras and rotation and rotator in them. But let us start with Clifford algebras.

In last years geometric algebras as Clifford algebras are called after David Hestenes (8) have gained popularity nearly in all areas of sciences where mathematics plays some role, see (9; 10; 11). What is the reason? Speaking in terms of our article, geometric algebra is more language of motions than language of objects. History of science shows that sciences have developed more as languages of objects. Thus, geometry from ancient times are considered as science of geometric figures, as language of geometric objects. Why ancients didn't consider geometry as language of geometric motions?

Let us consider very simple example. Area of rectangle is equal to that of parallelogram if the last is only some inclination of the first. To prove the fact, objective oriented geometry prove that two triangles are equal, i.e., that of left (in imaginable motion of inclination) and that of occupied. Thus, the proof is clearly geometrical in our traditional thinking. The same problem solved in geometrical algebra would use wedge product of two vectors $a \wedge b$, accordingly product doesn't depend from component of one vector that is parallel to other vector, i.e., only perpendicular component matters. We may interpret this in language of motions: the changing parallel component doesn't contribute to value of the product what concerns its scalar value, i.e., the value of parallelogram-form area the vectors span.

What conclusions may be made from this example? This example is too simple to see difference in approaches, but let us be confident in two aspects that can be captured from this example. First, geometric algebra approach is not so much object oriented but more motion oriented. Second, taking not one but many (even many many) problems we would considerably gain using just motion oriented approach, i.e., geometric algebra approach. Thus, geometric algebraic approach would be superior and prevailing in economy of effort immensely. But we must take in consideration that there may arise cases where object oriented approach would simply fail and motion oriented is the only working case. That is the case of quantum mechanics.

How to see that geometric algebra is motion oriented mathematics? We must admit that usual way how Clifford algebras are introduced doesn't show much that we should have to deal with motional rather than object oriented approach. Why so? Usually Clifford algebras are defined as graded algebras where grades arise due to geometric product defined as

$$AB = A \cdot B + A \wedge B$$

where $A \cdot B$ is scalar product part and $A \wedge B$ wedge product part of the product in general. Both parts pertain to different grades of algebra and thus graded algebra arises very naturally. But this starting point of Clifford algebra doesn't say much about the fact that algebra treats more "motions" than "objects". The essential nature of geometric algebra comes forth when we consider operator and operation of rotation (or versor more generally).

Rotation of arbitrary element of algebra x may be written $X \to RXR^{-1}$, where R is rotor, operator of rotation, that may be expressed in form $R = e^{b\theta/2}$, where b is bivector that spans the rotation and θ is angle of rotation. Here we see "mystical" number 2 or $1/2$, immanent to rotation. All know that rotator makes half of rotation in comparison with what is rotated; but why – nobody knows. We suggest way to solve this mystery with double rotation paradigm (considered further). Where to find second half of rotator? He/it moves in some other space? But spaces are not at all in principle. Let us reconstruct Motion in order to find missing half.

To do this we must consider reflection, i.e., one more versor in geometric algebra. In spatial relation rotation can be received changing sign of parallel projection with respect to normal to plane of reflection. But operational relation says us that reflection is half of rotation, because two reflections give always a rotation. More specifically, to rotate X to some state X' in plane determined by vectors m and n, we perform two successive reflections of X in planes with normals m and n correspondingly.

Let us speak about generalized rotation pertaining to Motion that would appear as operator rotator Rt with respect to rotation in spatial representation, but it must have second half of operator as reflector Rf (with $\pi/4$ spatial rotation in correspondence). When further we speak about double rotation we would assume this generalized rotator with 2π rotation in correspondence. Thus in spatial representation this generalized rotation has spatial double rotation in correspondence as well as usual reflection in correspondence. The crucial fact for physical laws in spatial representation is that as ground notion we must accept rotation what in spatial representation we perceive as double rotation. Thus we assume that behind these "visible" versors, double rotation and simple reflection stands some generalized rotation what pertains to Motion, or are aspects of this Motion.

Double rotation in quantum mechanics

Let us turn attention to role that group $SU(2)$ plays in quantum mechanics and Standard Model too. Group $SU(2)$ doubly covers group $O(3)$, and in this fact we see feature of Motion with its rotation that has double rotation in $O(3)$ in correspondence, i.e., with this we let us assume that Motion is more general "space" with its generalized rotation as physical space, and it must be taken as primal.

We must remember that Clifford algebra is essentially algebra of quantum mechanics making quantum mechanics language be more language of motion than object oriented. But Clifford algebra itself is double rotation algebra in Motion. So fact with groups $SU(2)$and $U(1) \times SU(2) \times SU(3)$ in quantum mechanics and in Standard Model is not only mechanical application of double rotation paradigm, but it says that this symmetry is primal "space" where quantum mechanical motion "lives in". Due to this fact we replace motion with Motion in quantum mechanical context. We believe that with this we succeed in excluding space and time aspects being direct present in description of reality. We try to replace them with Motion.

How to perceive this invisible Motion “space”? Possibly the use of geometric algebraic methods would be the only way to do this systematically, at least presently. Other ways would be accept Motion as closely related to Cognitum according approach in (12; 3; 13). Then connection mathematics with cognition might achieve new treatment. We come close to the position of authors Hu and Wu in (14) which state that spin is “more fundamental than spacetime itself, that is, spin is the “mind-pixel””.

To liberate quantum mechanics from its weirdness all quantum mechanical aspects should be found in macroworlds too. Exempli gratia, all features of quantum particle should be present as features of macro space time. Space time reconstructed using geometric algebra with some absent aspects should turn into some global “antiparticle”. This fact should be deducible from priority of Motion over visible space time. What we perceive in physical science as space time is some part of more general reality that Motion would reconstruct most easy, because what we need to get Motion we already have in appearance of quantum mechanics.

What should be Motion in physical science? In our approach we connect it with functionality of life. At least we are convinced that functionality of life is as wide and general as functionality where from life is built and in this sense we believe that there isn’t any more sense to refer to some other functionality behind the one of the life. From side of life what we reconstruct as Motion is more fundamental aspect of reality of space time. Of course, if physics want to remain in positions of undivided reductionism it may face problems to get over to ideas similar as in this article. Only all saving positivistic approach may would save physics this time too. Let it be so!

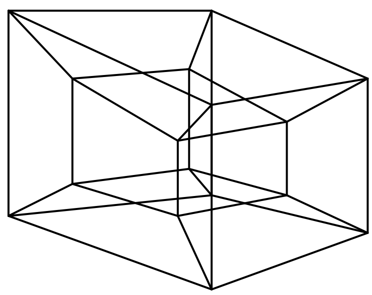

Fig.1

3D projection of tessaract. If we make it rotate, it visibly represents “spin of space” as two possible rotations. In Motion rotation shouldn’t be divided in two spin values, but remain one generalized rotation that has as its projections simple (double) rotation and reflection. The same or at least similar we get with simple experiment rotating some subject with respect a fixed point, see (15) p. 205 or (16) p. 43.

Hu and Wu approach to consciousness

Hu and Wu ground their approach mainly on the observation that "the probabilistic structure of quantum mechanics is due to the self-referential collapse of spin state that is contextual, non-local, non-computable and irreversible" (14). They accept quantum mechanical interpretation (17) of David Hestenes. They strongly hold on what they call existence of consciousness. For us the aspect is actual and interesting to see that spin in their approach becomes something called primordial and more fundamental than space time. They say: "Spin is the seat of consciousness and the linchpin between mind and the brain, that is, spin is the "mind-pixel."To overcome the absence of terminology they use these forms of expression „seat", „linchpin", „mind-pixel" which for contemporary science of level of quantum mechanics do not have any sense. We in place use notion of functionality of life to overcome the same hardness.

On quantum mechanical primacy what concerns space time

It would be hard to speak about whatever form of motion without space time in our traditional way of thinking given over us from times of Aristotle and even earlier. But just quantum mechanics with its new principles, e.g., in form of Heisenberg indeterminacy and particle wave duality, and so on gives us new approach to space time, or even to exclude space time. In what way? All what seams weird in quantum mechanics should be turned into reality of any or other form of motion in macrophysical appearance. We used for the form of motion new term Motion connecting it with life principle (*vita principalis*). Hu and Wu make stress on primacy of spin over space time, what in principle coincides with our approach of double rotation primacy. The crucial aspect of these and eventually other similar approaches is that quantum mechanics has already stepped over into new area where space time loses its primacy as background of all physical phenomena or stage where life of universe is staged on. We are invited to systematically apply this as general principle of primacy of quantum mechanics. Space time as background is questioned in (18; 19).
As visible aspect of space time reference, or Lorentzian reference of life, we see micro particles in probabilistic appearance what Einstein questioned or nicknamed "God is play dice? with us", but it is only due to generic features of space time being nothing primal, where primal is world of motions (behind space time) described in quantum mechanics mathematically with operator algebra (Clifford algebra). All other interpretations of QM, be they zitterbewegung or many worlds and so on, we refer to the same. They all are to be named interpretations of space time aspects in quantum mechanics, not quantum mechanics itself.
Triumph of quantum mechanics we recently observed in what occurred in connection with Hardy's paradox (20; 21; 22).
If we are allowed to provide future, we would guess that "all this" should come to a conclusion similar to the principle of Benjamin Lee Whorf (23), according which he states that language is not separable from cognition. Our guess would be that physics and mathematics should be indivisible from cognition too, (24). To be just to the past, we must acknowledge that Descartes was right in saying "Cogito ergo sum". What possibly may change (in time going on) is the interpretation and the depth of the meaning of these worlds. In the same time we don't believe that quantum mechanics should resolve problem of life, or describe or model life in whatever aspect. The only thing, quantum mechanics may help to determine life's primacy over reductional world picture in whatever aspect.

A tale

An old mathematician lived in ancient times who was summoned to king in order to narrate about his journey in far lands. The king asked: “What have you seen to relate about?”. “Oh, Highest” replied the old man “Many marvelous thing. But one among all I took with me.” “What it is, show me immediately”. “See, this is how to see all in the world. We may find out many ways to see possible movements and motions in the world around us, but adding them up and up there shouldn’t be end to this. But I learned in some far land that there suffices with one single motion to capture all known and not yet known taken together.” “What is this motion, show me?” – asked the king. “It is very simple and must be done by the person perceiving the world around him. He must simply turn around once, and this is all.” And the old man turned once around himself. “What do you want to say? Simple turn around tells you all in the world? You are overtired from the long journey, and either you have gone mad or you missed something to tell me mostly essential.” “Oh, Highest. I missed the crucial thing. Of course I forgot to tell something that matters immensely. There should be second turn around too.” And the old man turned around once more.

Conclusions

1) Geometric algebra methods should be used more where geometry itself should be derived as rotational/reflectional invariant motion with double rotation as base invariant in it.
2) Quantum mechanical laws of particles of microworld should be discovered as motions of physical macroworld.
3) Physics should be investigated more as reflection of world of motions rather than world of objects.
4) Quantum mechanical interpretations (at least part of them) may loose their necessity systematically turning attention more to investigation of world of motions than world of objects and phenomena.

We come close to the position of authors Hu and Wu in (14) which state that spin is “more fundamental than spacetime itself, that is, spin is the “mind-pixel””.

The main our conclusion would be to consider quantum mechanics itself, or its principles, as more primal than space time as background of physical laws in general.

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